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NEUTRINO MASSES, MIXING AND $(\beta\beta)_{0\nu}-$ DECAY

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ABSTRACT

The predictions for the effective Majorana mass $|<m>|$ in $(\beta\beta)_{0\nu}-$decay in the case of 3-ν mixing and massive Majorana neutrinos are reviewed. The physics potential of the experiments, searching for $(\beta\beta)_{0\nu}-$decay and having sensitivity to $|<m>| \gtrsim 0.01$ eV, for providing information on the type of the neutrino mass spectrum, on the absolute scale of neutrino masses and on the Majorana CP-violation phases in the PMNS neutrino mixing matrix is also discussed.

1. Introduction

The solar neutrino experiments Homestake, Kamiokande, SAGE, GALLEX/GNO, Super-Kamiokande (SK) and SNO [1,2,3,4], the data on atmospheric neutrinos obtained by the Super-Kamiokande (SK) experiment [5] and the results from the KamLAND reactor antineutrino experiment [6], provide very strong evidences for oscillations of flavour neutrinos driven by nonzero neutrino masses and neutrino mixing. The evidences for solar ν_e oscillations into active neutrinos $\nu_{\mu,\tau}$, in particular, were spectacularly reinforced by the first data from the SNO experiment [3] when combined with the data from the SK experiment [2], by the more recent SNO data [4], and by the first results of the KamLAND [6] experiment. Under the rather plausible assumption of CPT-invariance, the KamLAND data practically establishes [6] the large mixing angle (LMA) MSW solution as unique solution of the solar neutrino problem. This remarkable result brings us, after more than 30 years of research, initiated by the pioneer works of B. Pontecorvo [7] and the experiment of R. Davis et al. [8], very close to a complete understanding of the true cause of the solar neutrino problem.

The combined analyses of the available solar neutrino and KamLAND data, performed within the two-neutrino mixing hypothesis, identify two distinct solution subregions within the LMA solution region (see, e.g., [9,10,11]). The best fit values of the two-neutrino oscillation parameters - the solar neutrino mixing angle $\theta_\odot$ and the

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mass squared difference $\Delta m^2_\odot$, in the two sub-regions - low-LMA or LMA-I, and high-LMA or LMA-II, are given by (see, e.g., [9]):

$$\Delta m^{2\;\;I}_\odot = 7.3 \times 10^{-5}\ \mathrm{eV}^2, \qquad \tan^2\theta^I_\odot = 0.46, \tag{1}$$

$$\Delta m^{2\;\;II}_\odot = 1.5 \times 10^{-4}\mathrm{eV}^2, \qquad \tan^2\theta^{II}_\odot = 0.46. \tag{2}$$

The LMA-I solution is preferred statistically by the data. At 90% C.L. one finds [9]:

$$\Delta m^2_\odot \cong (5.6 - 17) \times 10^{-5}\ \mathrm{eV}^2, \quad \tan^2\theta_\odot \cong (0.32 - 0.72)\ . \tag{3}$$

The observed Zenith angle dependence of the multi-GeV $\mu-$like events in the Super-Kamiokande experiment unambiguously demonstrated the disappearance of the atmospheric ν_μ ($\bar\nu_\mu$) on distances $L \gtrsim 1000$ km. The Super-Kamiokande (SK) atmospheric neutrino data is best described in terms of dominant $\nu_\mu \to \nu_\tau$ ($\bar\nu_\mu \to \bar\nu_\tau$) oscillations with (almost) maximal mixing and neutrino mass squared difference of $|\Delta m^2_{\mathrm{A}}| \cong (1.8 - 4.0) \times 10^{-3}$ eV^2 (90% C.L.) [5]. According to the more recent combined analysis of the data from the SK and K2K experiments [12] one has:

$$2.1 \times 10^{-3}\,\mathrm{eV}^2 \lesssim |\Delta m^2_{\mathrm{A}}| \lesssim 3.3 \times 10^{-3}\,\mathrm{eV}^2\ , \qquad 90\%\ \mathrm{C.L.} \tag{4}$$

The interpretation of the solar and atmospheric neutrino, and of the KamLAND data in terms of neutrino oscillations requires the existence of 3-neutrino mixing in the weak charged lepton current (see, e.g., [13]):

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\,\nu_{j\mathrm{L}}\ . \tag{5}$$

Here ν_{lL}, $l = e, \mu, \tau$, are the three left-handed flavor neutrino fields, $\nu_{j\mathrm{L}}$ is the left-handed field of the neutrino ν_j having a mass m_j and U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix [14]. The PMNS mixing matrix U can be parametrized by three angles, θ_{atm}, $\theta_\odot$, and θ, and, depending on whether the massive neutrinos ν_j are Dirac or Majorana particles - by one or three CP-violating phases [15,16]. In the standard parametrization of U (see, e.g., [13]) the three mixing angles are denoted as θ_{12}, θ_{13} and θ_{23}:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13}e^{i\delta} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}e^{i\delta} \end{pmatrix} \mathrm{diag}(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}}) \tag{6}$$

where we have used the usual notations, $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, δ is the Dirac CP-violation phase and α_{21} and α_{31} are two Majorana CP-violation phases [15,16]. If we identify the two independent neutrino mass squared differences in this case, Δm^2_{21} and Δm^2_{31}, with the neutrino mass squared differences which induce the solar

and atmospheric neutrino oscillations, $\Delta m^2_{\odot} = \Delta m^2_{21} > 0$, $\Delta m^2_{\rm A} = \Delta m^2_{31}$, one has: $\theta_{12} = \theta_{\odot}$, $\theta_{23} = \theta_{\rm atm}$, and $\theta_{13} = \theta$. The angle θ is limited by the data from the CHOOZ and Palo Verde experiments [17,18]. The oscillations between flavour neutrinos are insensitive to the Majorana CP-violating phases α_{21}, α_{31} [15,19]. Information about these phases can be obtained, in principle, in the $(\beta\beta)_{0\nu}$-decay experiments [20,21,22,23,24,25] (see also [26,27,28]). Majorana CP-violating phases, and in particular, the phases α_{21} and/or α_{31}, might be at the origin of the baryon asymmetry of the Universe [29].

A 3-ν oscillation analysis of the CHOOZ data (in this case $\Delta m^2 = \Delta m^2_{\rm A}$) [30] led to the conclusion that for $\Delta m^2_{\odot} \lesssim 10^{-4}$ eV^2, the limits on $\sin^2\theta$ practically coincide with those derived in the 2-ν oscillation analysis in [17]. A combined 3-ν oscillation analysis of the solar neutrino, CHOOZ and the KamLAND data, performed under the assumption $\Delta m^2_{\odot} \ll |\Delta m^2_{\rm A}|$ (see, e.g., [13,31]), showed that [9]

$$\sin^2\theta < 0.05, \quad 99.73\% \text{ C.L.} \tag{7}$$

It was found [9] that the best-fit value of $\sin^2\theta$ lies in the interval $\sin^2\theta \cong (0.00-0.01)$.

Somewhat better limits on $\sin^2\theta$ than the existing one can be obtained in the MINOS experiment [32]. Various options are being currently discussed (experiments with off-axis neutrino beams, more precise reactor antineutrino and long baseline experiments, etc., see, e.g., [33,34]) of how to improve by at least a factor of 5 or more, i.e., to values of ~ 0.01 or smaller, the sensitivity to $\sin^2\theta$.

Let us note that the atmospheric neutrino and K2K data do not allow one to determine the sign of $\Delta m^2_{\rm A}$. This implies that if we identify $\Delta m^2_{\rm A}$ with Δm^2_{31} in the case of 3-neutrino mixing, one can have $\Delta m^2_{31} > 0$ or $\Delta m^2_{31} < 0$. The two possibilities correspond to two different types of neutrino mass spectrum: with normal hierarchy, $m_1 < m_2 < m_3$, and with inverted hierarchy, $m_3 < m_1 < m_2$. We will use the terms *normal hierarchical (NH)* and *inverted hierarchical (IH)* for the two types of spectra in the case of strong inequalities between the neutrino masses, if $m_1 \ll m_2 \ll m_3$ and [c] $m_3 \ll m_1 < m_2$, respectively. The spectrum can also be of *quasi-degenerate (QD)* type: $m_1 \cong m_2 \cong m_3$, $m^2_{1,2,3} >> |\Delta m^2_{\rm A}|$.

The sign of $\Delta m^2_{\rm A}$ can be determined in very long base-line neutrino oscillation experiments at neutrino factories (see, e.g., [36]), and, e.g, using combined data from long base-line oscillation experiments at the JHF facility and with off-axis neutrino beams [37]. Under certain rather special conditions it might be determined also in experiments with reactor $\bar{\nu}_e$ [38,35].

[c]This definition of the IH spectrum corresponds to a convention we will call A (see, e.g., [35]), in which the neutrino masses are not ordered in magnitude according to their index number. We can also always number the neutrinos with definite mass in such a way that [20,30] $m_1 < m_2 < m_3$. In this convention called B [35] the IH spectrum corresponds to $m_1 << m_2 \cong m_3$. We will use convention B in our further analysis.

As is well-known, neutrino oscillation experiments allow one to determine differences of squares of neutrino masses, but not the absolute values of the masses. Information on the absolute values of neutrino masses of interest can be derived in the ^{3}H β-decay experiments studying the electron spectrum [39,40,41] and from cosmological and astrophysical data (see, e.g., ref. [42,43,44]). The currently existing most stringent upper bounds on the electron (anti-)neutrino mass $m_{\bar{\nu}_e}$ were obtained in the Troitzk [40] and Mainz [41] ^{3}H β-decay experiments and read:

$$m_{\bar{\nu}_e} < 2.2 \text{ eV} \quad (95\% \; C.L.). \tag{8}$$

We have $m_{\bar{\nu}_e} \cong m_{1,2,3}$ in the case of QD neutrino mass spectrum. The KATRIN ^{3}H β-decay experiment [41] is planned to reach a sensitivity to $m_{\bar{\nu}_e} \sim (0.20 - 0.35)$ eV. The data of the WMAP experiment on the cosmic microwave background radiation was used to obtain an upper limit on the sum of the neutrino masses [43]:

$$\sum_j m_j < 0.70 \;\; \text{eV} \quad (95\% \; C.L.). \tag{9}$$

A conservative estimate of all the uncertainties related to the derivation of this result (see, e.g., [45]) lead to a less stringent upper limit at least by a factor of ~ 1.5 and possibly by a factor of ~ 3. The WMAP and future PLANCK experiments can be sensitive to [42] $\sum_j m_j \cong 0.4$ eV. Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and PLANCK experiments may allow one to determine $(m_1 + m_2 + m_3)$ with an uncertainty of [44] $\delta \sim 0.04$ eV.

After the spectacular experimental progress made in the last two years or so in the studies of neutrino oscillations, further understanding of the structure of the neutrino masses and mixing, of their origins and of the status of the CP-symmetry in the lepton sector requires a large and challenging program of research to be pursued in neutrino physics. The main goals of such a research program should include:

– High precision determination of neutrino mixing parameters which control the solar and the dominant atmospheric neutrino oscillations, $\Delta m^2_{\odot}$, $\theta_{\odot}$, and $\Delta m^2_{\rm A}$, θ_{atm}.

– Measurement of, or improving by at least a factor of (5 - 10) the existing upper limit on, the value of the only small mixing angle θ $(= \theta_{13})$ in the PMNS matrix U.

– Determination of the type of the neutrino mass spectrum (normal hierarchical, or inverted hierarchical, or quasi-degenerate).

– Determining or obtaining significant constraints on the absolute scale of neutrino masses, or on the lightest neutrino mass.

– Determining the nature of massive neutrinos which can be Dirac or Majorana particles.

– Establish whether the CP-symmetry is violated in the lepton sector a) due to the Dirac phase δ, and/or b) due to the Majorana phases α_{21} and α_{31} if the massive neutrinos are Majorana particles.

– Searching with increased sensitivity for possible manifestations, other than flavour neutrino oscillations, of the non-conservation of the individual lepton charges L_l, $l = e, \mu, \tau$, such as $\mu \to e + \gamma$, $\tau \to \mu + \gamma$, etc. decays.
– Understanding at fundamental level the mechanism giving rise to the neutrino masses and mixing and to the L_l-non-conservation, i.e., finding the Theory of neutrino mixing. Progress in the theory of neutrino mixing might also lead, in particular, to a better understanding of the possible relation between CP-violation in the lepton sector at low energies and the generation of the baryon asymmetry of the Universe.

Obviously, the successful realization of the experimental part of this program of research would be a formidable task and would require most probably (10 - 15) years.

In the present article we will review the potential contribution the studies of neutrinoless double beta ($(\beta\beta)_{0\nu}-$) decay of certain even-even nuclei, $(A,Z) \to (A,Z+2) + e^- + e^-$, can make to the program of research outlined above. The $(\beta\beta)_{0\nu}$-decay is allowed if the neutrinos with definite mass are Majorana particles (for reviews see, e.g., [46,47]). Let us recall that the nature - Dirac or Majorana, of the massive neutrinos ν_j, is related to the fundamental symmetries of the particle interactions. The neutrinos ν_j will be Dirac fermions if the particle interactions conserve some lepton charge, which could be, e.g., the total lepton charge L. The neutrinos with definite mass can be Majorana particles if there does not exist any conserved lepton charge. As is well-known, the massive neutrinos are predicted to be of Majorana nature by the see-saw mechanism of neutrino mass generation [48], which also provides a very attractive explanation of the smallness of the neutrino masses and - through the leptogenesis theory [29], of the observed baryon asymmetry of the Universe.

If the massive neutrinos ν_j are Majorana fermions, processes in which the total lepton charge L is not conserved and changes by two units, such as $K^+ \to \pi^- + \mu^+ + \mu^+$, $\mu^+ + (A,Z) \to (A,Z+2) + \mu^-$, etc., should exist. The process most sensitive to the possible Majorana nature of the massive neutrinos ν_j is the $(\beta\beta)_{0\nu}-$decay (see, e.g., [46]). If the $(\beta\beta)_{0\nu}$-decay is generated *only by the (V-A) charged current weak interaction via the exchange of the three Majorana neutrinos ν_j and the latter have masses not exceeding few MeV*, which will be assumed to hold throughout this article, the dependence of the $(\beta\beta)_{0\nu}$-decay amplitude $A(\beta\beta)_{0\nu}$ on the neutrino mass and mixing parameters factorizes in the effective Majorana mass $<\!m\!>$ (see, e.g., [46,47]):

$$A(\beta\beta)_{0\nu} \sim \,<\!m\!>\, M \;, \tag{10}$$

where M is the corresponding nuclear matrix element (NME) and

$$|<\!m\!>| \;= \left| m_1 |U_{\mathrm{e}1}|^2 + m_2 |U_{\mathrm{e}2}|^2 \; e^{i\alpha_{21}} + m_3 |U_{\mathrm{e}3}|^2 \; e^{i\alpha_{31}} \right| \;, \tag{11}$$

α_{21} and α_{31} being the two Majorana CP-violating phases of the PMNS matrix [d]

[d] We assume that $m_j > 0$ and that the fields of the Majorana neutrinos ν_j satisfy the Majorana condition: $C(\bar{\nu}_j)^T = \nu_j,\; j = 1,2,3$, where C is the charge conjugation matrix.

[15,16]. Let us note that if CP-invariance holds, one has [49] $\alpha_{21} = k\pi$, $\alpha_{31} = k'\pi$, where $k, k' = 0, 1, 2, ...$ In this case

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1 \tag{12}$$

represent the relative CP-parities of the Majorana neutrinos ν_1 and ν_2, and ν_1 and ν_3, respectively. It follows from eq. (11) that the measurement of $|{<}m{>}|$ will provide information, in particular, on the neutrino masses. As eq. (10) indicates, the observation of the $(\beta\beta)_{0\nu}$-decay of a given nucleus and the measurement of the corresponding half life-time, would allow one to determine $|{<}m{>}|$ only if the value of the relevant NME M is known with a relatively small uncertainty.

The experimental searches for $(\beta\beta)_{0\nu}-$decay have a long history (see, e.g., [47]). Rather stringent upper bounds on $|{<}m{>}|$ have been obtained in the ^{76}Ge experiments by the Heidelberg-Moscow collaboration [50]:

$$|{<}m{>}| < 0.35 \text{ eV}, \quad 90\% \text{ C.L.} \tag{13}$$

Taking into account a factor of 3 uncertainty associated with the calculation of the relevant nuclear matrix element [47], we get [e]

$$|{<}m{>}| < (0.35 - 1.05) \text{ eV}, \quad 90\% \text{ C.L.} \tag{14}$$

The IGEX collaboration has obtained [53]:

$$|{<}m{>}| < (0.33 \div 1.35) \text{ eV}, \quad 90\% \text{ C.L.} \tag{15}$$

Higher sensitivity to $|{<}m{>}|$ is planned to be reached in several $(\beta\beta)_{0\nu}$-decay experiments of a new generation. The NEMO3 experiment [54] with ^{100}M and ^{82}Se, which began to take data in July of 2002, and the cryogenics detector CUORICINO [56], which uses ^{130}Te and is already operative, are expected to reach a sensitivity to values of $|{<}m{>}| \sim 0.2$ eV. The first preliminary results from these two experiments were announced recently [54,55] and respectively read (90% C.L.): $|{<}m{>}| < (1.2 - 2.7)$ eV and $|{<}m{>}| < (0.7 - 1.7)$ eV. Up to an order of magnitude better sensitivity, i.e., to $|{<}m{>}| \cong 2.7 \times 10^{-2}$ eV, 1.5×10^{-2} eV, 5.0×10^{-2} eV, 2.5×10^{-2} eV and 3.6×10^{-2} eV is planned to be achieved in the CUORE, GENIUS, EXO, MAJORANA and MOON experiments [56] [f] with ^{130}Te, ^{76}Ge, ^{136}Xe, ^{76}Ge and ^{100}Mo, respectively. Additional high sensitivity experiments with ^{136}Xe - XMASS [58], and with ^{48}Ca - CANDLES [59], are also being considered.

[e] Evidences for $(\beta\beta)_{0\nu}$-decay taking place with a rate corresponding to 0.11 eV $\leq |{<}m{>}| \leq 0.56$ eV (95% C.L.) are claimed to have been obtained in [51]. The results announced in [51] have been criticized in [52].

[f] The quoted sensitivities correspond to values of the relevant NME from ref. [57].

As we will discuss in what follows, the studies of $(\beta\beta)_{0\nu}-$decay and a measurement of a nonzero value of $|<m>| \gtrsim$ few 10^{-2} eV:
– Can establish the Majorana nature of massive neutrinos. The $(\beta\beta)_{0\nu}$-decay experiments are presently the only feasible experiments capable of doing that (see [46]).
– Can give information on the type of neutrino mass spectrum [60,61,21,24,23,62,63,64]. More specifically, a measured value of $|<m>| \sim$ few $\times 10^{-2}$ eV can provide, in particular, unique constraints on, or even can allow one to determine, the type of the neutrino mass spectrum if $\nu_{1,2,3}$ are Majorana particles [63].
– Can provide also unique information on the absolute scale of neutrino masses, or on the lightest neutrino mass (see, e.g., [60,61,24,62]).
– With additional information from other sources (^{3}H β-decay experiments or cosmological and astrophysical data and considerations) on the absolute neutrino mass scale, the $(\beta\beta)_{0\nu}-$decay experiments can provide unique information on the Majorana CP-violation phases α_{21} and α_{31} [20,21,23,24,25].

2. Predictions for the Effective Majorana Mass

The predicted value of $|<m>|$ depends in the case of $3-\nu$ mixing on [65] (see also [60,21]): i) $\Delta m^2_{\rm A}$, ii) $\theta_\odot$ and $\Delta m^2_\odot$, iii) the lightest neutrino mass, and on iv) the mixing angle θ. Using the convention (B) in which always $m_1 < m_2 < m_3$, one has $\Delta m^2_{\rm A} \equiv \Delta m^2_{31}$, and $m_3 = \sqrt{m_1^2 + \Delta m^2_{\rm A}}$, while either $\Delta m^2_\odot \equiv \Delta m^2_{21}$ (normal mass hierarchy) or $\Delta m^2_\odot \equiv \Delta m^2_{32}$ (inverted mass hierarchy). In the first case we have $m_2 = \sqrt{m_1^2 + \Delta m^2_\odot}$, $|U_{\rm e1}|^2 = \cos^2\theta_\odot(1-|U_{\rm e3}|^2)$, $|U_{\rm e2}|^2 = \sin^2\theta_\odot(1-|U_{\rm e3}|^2)$, and $|U_{\rm e3}|^2 \equiv \sin^2\theta$, while in the second $m_2 = \sqrt{m_1^2 + \Delta m^2_{\rm A} - \Delta m^2_\odot}$, $|U_{\rm e2}|^2 = \cos^2\theta_\odot(1-|U_{\rm e1}|^2)$, $|U_{\rm e3}|^2 = \sin^2\theta_\odot(1-|U_{\rm e1}|^2)$, and $|U_{\rm e1}|^2 \equiv \sin^2\theta$. The two possibilities for $\Delta m^2_\odot$ correspond also to the two different *hierarchical* types of neutrino mass spectrum — the *normal hierarchical (NH)*, $m_1 << m_2 << m_3$, and the *inverted hierarchical (IH)*, $m_1 << m_2 \cong m_3$, respectively. Let us recall that in the case of *quasi-degenerate (QD)* neutrino mass spectrum we have $m_1 \cong m_2 \cong m_3$, $m^2_{1,2,3} >> \Delta m^2_{\rm A}$. For the allowed ranges of values of $\Delta m^2_\odot$ and $\Delta m^2_{\rm A}$ [5], the NH (IH) spectrum corresponds to $m_1 \lesssim 10^{-3}$ (2×10^{-2}) eV, while one has a QD spectrum if $m_{1,2,3} \cong m_{\bar{\nu}_e} > 0.20$ eV. For m_1 lying in the interval between $\sim 10^{-3}$ (2×10^{-2}) eV and 0.20 eV, the neutrino mass spectrum is with partial normal (inverted) hierarchy (see, e.g., [21]).

Given $\Delta m^2_\odot$, $\Delta m^2_{\rm A}$, $\theta_\odot$ and $\sin^2\theta$, the value of $|<m>|$ depends strongly on the type of the neutrino mass spectrum as well as on the values of the two Majorana CP-violation phases of the PMNS matrix, α_{21} and α_{31} (see eq. (11)). Let us note that in the case of QD spectrum, $m_1 \cong m_2 \cong m_3$, $m^2_{1,2,3} \gg \Delta m^2_{\rm A}, \Delta m^2_\odot$, $|<m>|$ is essentially independent on $\Delta m^2_{\rm A}$ and $\Delta m^2_\odot$, and the two possibilities, $\Delta m^2_\odot \equiv \Delta m^2_{21}$ and $\Delta m^2_\odot \equiv \Delta m^2_{32}$, lead *effectively* to the same predictions for $|<m>|$ [g].

[g] This statement is valid, within the convention $m_1 < m_2 < m_3$ we are using, as long as there are

2.1. Normal Hierarchical Neutrino Mass Spectrum

In the case of NH neutrino mass spectrum one has $m_2 \cong \sqrt{\Delta m^2_\odot}$, $m_3 \cong \sqrt{\Delta m^2_{\rm atm}}$, $|U_{\rm e3}|^2 \equiv \sin^2\theta$, and correspondingly

$$|{<}\,m\,{>}| \;= \left|(m_1 \cos^2\theta_\odot + e^{i\alpha_{21}}\sqrt{\Delta m^2_\odot}\,\sin^2\theta_\odot)\cos^2\theta + \sqrt{\Delta m^2_{\rm A}}\,\sin^2\theta\; e^{i\alpha_{31}}\right| \quad (16)$$

$$\simeq \left|\sqrt{\Delta m^2_\odot}\,\sin^2\theta_\odot\cos^2\theta + \sqrt{\Delta m^2_{\rm A}}\,\sin^2\theta e^{i(\alpha_{31}-\alpha_{21})}\right| \quad (17)$$

where we have neglected the term $\sim m_1$ in eq. (17). Although in this case one of the three massive Majorana neutrinos effectively "decouples" and does not give a contribution to $|{<}\,m\,{>}|$, the value of $|{<}\,m\,{>}|$ still depends on the Majorana CP-violation phase $\alpha_{32} = \alpha_{31} - \alpha_{21}$. This reflects the fact that in contrast to the case of massive Dirac neutrinos (or quarks), CP-violation can take place in the mixing of only two massive Majorana neutrinos [15].

Since, as it follows from eqs. (3) and (4), we have $\sqrt{\Delta m^2_\odot} \lesssim 1.3\times10^{-2}$ eV, $\sin^2\theta_\odot \lesssim 0.42$, $\sqrt{\Delta m^2_{\rm A}} \lesssim 5.5\times10^{-2}$ eV, and the largest neutrino mass enters into the expression for $|{<}\,m\,{>}|$ with the factor $\sin^2\theta < 0.05$, the predicted value of $|{<}\,m\,{>}|$ is below 10^{-2} eV: for $\sin^2\theta = 0.05$ (0.01) one finds $|{<}\,m\,{>}| \lesssim 0.0086$ (0.0066) eV. Using the best fit values of the indicated neutrino oscillation parameters we get even smaller values for $|{<}\,m\,{>}|$, $|{<}\,m\,{>}| \lesssim 0.0059$ (0.0039) eV (see Tables 1 and 2). Actually, it follows from eq. (16) and the allowed ranges of values of $\Delta m^2_\odot$, $\Delta m^2_{\rm A}$, $\sin^2\theta_\odot$, $\sin^2\theta$ as well as of the lightest neutrino mass m_1 and the CP-violation phases α_{21} and α_{31}, that in the case of NH spectrum there can be a complete cancellation between the contributions of the three terms in eq. (16) and one can have [24] $|{<}\,m\,{>}| = 0$.

2.2. Inverted Hierarchical Spectrum

One has for the IH neutrino mass spectrum (see, e.g. [21]) $m_2 \cong m_3 \cong \sqrt{\Delta m^2_{\rm A}}$, $|U_{\rm e1}|^2 \equiv \sin^2\theta$. Neglecting $m_1\sin^2\theta$ in eq. (11), we find [20,60,21]:

$$|{<}\,m\,{>}| \;\cong \sqrt{\Delta m^2_{\rm A}}\,\cos^2\theta\sqrt{1-\sin^2 2\theta_\odot\sin^2\left(\frac{\alpha_{32}}{2}\right)}. \quad (18)$$

Even though one of the three massive Majorana neutrinos "decouples", the value of $|{<}\,m\,{>}|$ depends on the Majorana CP-violating phase $\alpha_{32} \equiv (\alpha_{31}-\alpha_{21})$. Obviously,

$$\sqrt{\Delta m^2_{\rm A}}\,\cos^2\theta|\;\cos 2\theta_\odot|\;\le\; |{<}\,m\,{>}|\;\le \sqrt{\Delta m^2_{\rm A}}\,\cos^2\theta. \quad (19)$$

no independent constraints on the CP-violating phases α_{21} and α_{31} which enter into the expression for $|{<}\,m\,{>}|$. In the case of spectrum with normal hierarchy, $|{<}\,m\,{>}|$ depends primarily on α_{21} ($|U_{\rm e3}|^2 \ll 1$), while if the spectrum is with inverted hierarchy, $|{<}\,m\,{>}|$ will depend essentially on $\alpha_{31}-\alpha_{21}$ ($|U_{\rm e1}|^2 \ll 1$).

The upper and the lower limits correspond respectively to the CP-conserving cases $\alpha_{32} = 0$, or $\alpha_{21} = \alpha_{31} = 0, \pm\pi$, and $\alpha_{32} = \pm\pi$, or $\alpha_{21} = \alpha_{31} + \pi = 0, \pm\pi$. Most remarkably, since according to the solar neutrino and KamLAND data $\cos 2\theta_\odot \sim (0.35 - 0.40)$, we get a significant lower limit on $|<m>|$, typically exceeding 10^{-2} eV, in this case [63,24] (Tables 1 and 2). Using, e.g., the best fit values of $\Delta m^2_{\rm A}$ and $\tan^2\theta_\odot$ one finds: $|<m>| \gtrsim 0.018$ eV. The maximal value of $|<m>|$ is determined by $\Delta m^2_{\rm A}$ and can reach, as it follows from eq. (4), $\Delta m^2_{\rm A} \sim 6 \times 10^{-2}$ eV. The indicated values of $|<m>|$ are within the range of sensitivity of the next generation of $(\beta\beta)_{0\nu}$-decay experiments.

The expression for $|<m>|$, eq. (18), permits to relate the value of $\sin^2(\alpha_{31} - \alpha_{21})/2$ to the experimentally measured quantities [20,21] $|<m>|$, $\Delta m^2_{\rm atm}$ and $\sin^2 2\theta_\odot$:

$$\sin^2\frac{\alpha_{31} - \alpha_{21}}{2} \cong \left(1 - \frac{|<m>|^{\,2}}{\Delta m^2_{\rm A}\, \cos^4\theta}\right)\frac{1}{\sin^2 2\theta_\odot}. \tag{20}$$

A more precise determination of $\Delta m^2_{\rm A}$ and $\theta_\odot$ and a sufficiently accurate measurement of $|<m>|$ could allow one to get information about the value of $(\alpha_{31} - \alpha_{21})$, provided the neutrino mass spectrum is of the IH type.

2.3. Three Quasi-Degenerate Neutrinos

In this case it is convenient to introduce $m_0 \equiv m_1 \cong m_2 \cong m_3$, $m_0^2 \gg \Delta m^2_{\rm A}$, $m_0 \gtrsim 0.20$ eV. The mass m_0 effectively coincides with the electron (anti-)neutrino mass $m_{\bar{\nu}_e}$ measured in the ^{3}H β-decay experiments: $m_0 = m_{\bar{\nu}_e}$. Thus, $m_0 < 2.2$ eV , or if we use a conservative cosmological upper limit [45] $m_0 < 0.7$ eV. The QD neutrino mass spectrum is realized for values of m_0, which can be measured in the ^{3}H $\beta-$decay experiment KATRIN.

The effective Majorana mass $|<m>|$ is given by

$$|<m>| \cong m_0 \left|(\cos^2\theta_\odot + \sin^2\theta_\odot e^{i\alpha_{21}})\, \cos^2\theta + e^{i\alpha_{31}}\sin^2\theta\right| \tag{21}$$

$$\cong m_0 \left|\cos^2\theta_\odot + \sin^2\theta_\odot e^{i\alpha_{21}}\right| = m_0\,\sqrt{1 - \sin^2 2\theta_\odot \sin^2\left(\frac{\alpha_{21}}{2}\right)}. \tag{22}$$

Similarly to the case of IH spectrum, one has:

$$m_0\ |\cos 2\theta_\odot| \ \lesssim |<m>| \ \lesssim m_0. \tag{23}$$

For $\cos 2\theta_\odot \sim (0.35 - 0.40)$ favored by the solar neutrino and the KamLAND data one finds a non-trivial lower limit of on $|<m>|$, $|<m>| \gtrsim (0.06 - 0.07)$ eV. Using the conservative cosmological upper bound on the sum of neutrino masses we get $|<m>| \lesssim 0.70$ eV. Also in this case one can obtain, in principle, a direct information on one CP-violation phase from the measurement of $|<m>|$, m_0 and $\sin^2 2\theta_\odot$:

$$\sin^2\frac{\alpha_{21}}{2} \cong \left(1 - \frac{|<m>|^{\,2}}{m_0^2}\right)\frac{1}{\sin^2 2\theta_\odot}. \tag{24}$$

The specific features of the predictions for $|<m>|$ in the cases of the three types of neutrino mass spectrum discussed above are evident in Figs. 1 and 2, where the dependence of $|<m>|$ on m_1 for the two possible sub-regions of the LMA solution region - LMA-I and LMA-II, is shown. If $\Delta m^2_{\odot} = \Delta m^2_{21}$, for instance, which corresponds to a spectrum with normal hierarchy, $|<m>|$ can lie anywhere between 0 and the presently existing upper limits, given by eqs. (14) and (15). This conclusion does not change even under the most favorable conditions for the determination of $|<m>|$, namely, even when $\Delta m^2_{\rm atm}$, $\Delta m^2_{\odot}$, $\theta_{\odot}$ and θ are known with negligible uncertainty, as Fig. 1, upper left panel, and Fig. 2, upper panel, indicate.

3. Constraining the Lightest Neutrino Mass

If the $(\beta\beta)_{0\nu}$-decay of a given nucleus will be observed, it would be possible to determine the value of $|<m>|$ from the measurement of the associated life-time of the decay. This would require the knowledge of the nuclear matrix element of the process. At present there exist large uncertainties in the calculation of the $(\beta\beta)_{0\nu}$-decay nuclear matrix elements (see, e.g., [47,66]). This is reflected, in particular, in the factor of ~ 3 uncertainty in the upper limit on $|<m>|$, which is extracted from the experimental lower limits on the $(\beta\beta)_{0\nu}$-decay half life-time of ^{76}Ge. The observation of a $(\beta\beta)_{0\nu}$-decay of one nucleus is likely to lead to the searches and eventually to observation of the decay of other nuclei. One can expect that such a progress, in particular, will help to solve the problem of the sufficiently precise calculation of the nuclear matrix elements for the $(\beta\beta)_{0\nu}$-decay.

In this Section we consider briefly the information that future $(\beta\beta)_{0\nu}$-decay and/or ^{3}H $\beta-$decay experiments can provide on the lightest neutrino mass m_1, without taking into account the possible effects of the currently existing uncertainties in the evaluation of the $(\beta\beta)_{0\nu}$-decay nuclear matrix elements.

An experimental upper limit on $|<m>|$, $|<m>| < |<m>|_{\ exp}$, will determine a maximal value of m_1, $m_1 < (m_1)_{max}$ in the case of normal mass hierarchy, $\Delta m^2_{\odot} \equiv \Delta m^2_{21}$ (Figs. 1, 2). For the QD spectrum, for instance, we have $m_1 \gg \Delta m^2_{\odot}$, $\Delta m^2_{\rm A}$, and up to corrections $\sim \Delta m^2_{\odot} \sin^2\theta_{\odot}/(2m_1^2)$ and $\sim \Delta m^2_{\rm A} \sin^2\theta/(2m_1^2)$ one finds [24,67]:

$$(m_1)_{max} \cong \frac{|<m>|_{\ exp}}{\left|\cos 2\theta_{\odot}\cos^2\theta - \sin^2\theta\right|} \ . \tag{25}$$

We get similar results in the case of inverted mass hierarchy, $\Delta m^2_{\odot} \equiv \Delta m^2_{32}$, provided the experimental upper limit $|<m>|_{\ exp}$ is larger than the minimal value of $|<m>|$, $|<m>|^{\ ph}_{\ min}$ (Figs. 1, 2), predicted by taking into account all uncertainties in the values of the relevant input parameters ($\Delta m^2_{\rm A}$, $\Delta m^2_{\odot}$, $\theta_{\odot}$, etc.). If $|<m>|_{\ exp} < |<m>|^{\ ph}_{\ min}$, then either i) the neutrino mass spectrum is not of the inverted hierarchy type, or ii) there exist contributions to the $(\beta\beta)_{0\nu}$-decay rate other than due to the light Majorana neutrino exchange (see, e.g., [68]) that partially cancel

the contribution from the Majorana neutrino exchange. The indicated result might also suggest that the massive neutrinos are Dirac particles.

A measurement of $|<m>| = (|<m>|)_{exp} \gtrsim 0.02$ eV if $\Delta m^2_\odot \equiv \Delta m^2_{21}$, and of $|<m>| = (|<m>|)_{exp} \gtrsim \sqrt{\Delta m^2_{\rm A}} \cos^2\theta$ in the case of $\Delta m^2_\odot \equiv \Delta m^2_{32}$, would imply that $m_1 \gtrsim 0.02$ eV and $m_1 \gtrsim 0.04$ eV, respectively, and thus a neutrino mass spectrum with partial hierarchy or of the QD type [21] (Figs. 1, 2). The lightest neutrino mass will be constrained to lie in a rather narrow interval, $(m_1)_{min} \leq m_1 \leq (m_1)_{max}$ [h]. The limiting values of m_1 correspond to the case of CP-conservation. For $\Delta m^2_\odot \ll m_1^2$, (i.e., for $\Delta m^2_\odot \lesssim 10^{-4}$ eV^2), as can be shown [24], we have $(m_1)_{min} \cong (|<m>|)_{exp}$ for $\Delta m^2_\odot \equiv \Delta m^2_{21}$, and $\sqrt{((m_1)_{min})^2 + \Delta m^2_{\rm A}} \cong (|<m>|)_{exp}$ for $\Delta m^2_\odot \equiv \Delta m^2_{32}$.

A measured value of $|<m>|$ satisfying $(|<m>|)_{exp} < (|<m>|)_{max}$, where, e.g., in the case of a QD spectrum $(|<m>|)_{max} \cong m_1 \cong m_{\bar{\nu}_e}$, would imply that at least one of the two CP-violating phases is different from zero : $\alpha_{21} \neq 0$ or $\alpha_{31} \neq 0$ [i].

If the measured value of $|<m>|$ lies between the minimal and maximal values of $|<m>|$, predicted in the case of inverted *hierarchical* spectrum,

$$|<m>|_{\pm} = \left|\sqrt{\Delta m^2_{\rm A} - \Delta m^2_\odot} \cos^2\theta_\odot \pm \sqrt{\Delta m^2_{\rm A}} \sin^2\theta_\odot\right| \cos^2\theta, \qquad (26)$$

m_1 again would be limited from above, but we would have $(m_1)_{min} = 0$ (Figs. 1, 2).

A measured value of $m_{\bar{\nu}_e}$, $(m_{\bar{\nu}_e})_{exp} \gtrsim 0.20$ eV, satisfying $(m_{\bar{\nu}_e})_{exp} > (m_1)_{max}$, where $(m_1)_{max}$ is determined from the upper limit on $|<m>|$ in the case the $(\beta\beta)_{0\nu}$-decay is not observed, might imply that the massive neutrinos are Dirac particles. If $(\beta\beta)_{0\nu}$-decay has been observed and $|<m>|$ measured, the inequality $(m_{\bar{\nu}_e})_{exp} > (m_1)_{max}$, with $(m_1)_{max}$ determined from the measured value of $|<m>|$, would lead to the conclusion that there exist contribution(s) to the $(\beta\beta)_{0\nu}$-decay rate other than due to the light Majorana neutrino exchange (see, e.g., [68] and the references quoted therein) that partially cancels the contribution from the Majorana neutrino exchange.

4. Determining the Type of Neutrino Mass Spectrum

The possibility to distinguish between the three different types of neutrino mass spectrum - NH, IH and QD, depends on the allowed ranges of values of $|<m>|$ for the three spectra. More specifically, it is determined by the maximal values of $|<m>|$ in the cases of NH and IH spectra, $|<m>|^{\rm NH}_{\rm max}$ and $|<m>|^{\rm IH}_{\rm max}$, and by the minimal values of $|<m>|$ for the IH and QD spectra, $|<m>|^{\rm IH}_{\rm min}$ and $|<m>|^{\rm QD}_{\rm min}$. These can be derived from eqs. (16), (19) and (21) and correspond to CP-conserving values of the Majorana phases [64] α_{21} and α_{31}. The minimal value $|<m>|^{\rm QD}_{\rm min}$ scales to a good approximation with m_0 and thus is reached for $m_0 = 0.2$ eV.

[h]Analytic expressions for $(m_1)_{min}$ and $(m_1)_{max}$ are given in [24].

[i]Let us note that, in general, the knowledge of the value of $|<m>|$ alone will not allow to distinguish the case of CP-conservation from that of CP-violation.

In Tables 1 and 2 (taken from [64]) we show the values of i) $|<m>|_{\rm max}^{\rm NH}$, ii) $|<m>|_{\rm min}^{\rm IH}$, and iii) $|<m>|_{\rm min}^{\rm QD}$ ($m_0 = 0.2$ eV), calculated for the best-fit and the 90% C.L. allowed ranges of values of $\tan^2\theta_\odot$ and $\Delta m^2_\odot$ in the LMA solution region. In Table 3 (from [64]) we give the same quantities, $|<m>|_{\rm max}^{\rm NH}$, $|<m>|_{\rm min}^{\rm IH}$ and $|<m>|_{\rm min}^{\rm QD}$, calculated using the best-fit values of the neutrino oscillation parameters, including 1 s.d. (3 s.d.) "prospective" uncertainties [j] of 5% (15%) on $\tan^2\theta_\odot$ and $\Delta m^2_\odot$, and of 10% (30%) on $\Delta m^2_{\rm A}$. As is evident from Tables 1 - 3, the possibility of determining the type of the neutrino mass spectrum if $|<m>|$ is found to be nonzero in the $(\beta\beta)_{0\nu}$-decay experiments of the next generation, depends crucially on the precision with which $\Delta m^2_{\rm A}$, $\theta_\odot$, $\Delta m^2_\odot$, $\sin^2\theta$ and $|<m>|$ will be measured. It depends also crucially on the values of $\theta_\odot$ and of $|<m>|$. The precision itself of the measurement of $|<m>|$ in the next generation of $(\beta\beta)_{0\nu}$- decay experiments, given the latter sensitivity limits of $\sim (1.5 - 5.0) \times 10^{-2}$ eV, depends on the value of $|<m>|$. The precision in the measurements of $\tan^2\theta_\odot$ and $\Delta m^2_\odot$ used in order to derive the numbers in Table 3 can be achieved, e.g., in the solar neutrino experiments and/or in the experiments with reactor $\bar{\nu}_e$ [69,35]. If $\Delta m^2_{\rm A}$ lies in the interval $\Delta m^2_{\rm A} \cong (2.0 - 5.0) \times 10^{-3}$ eV^2, as is suggested by the current data [5,12], its value will be determined with a $\sim 10\%$ error (1 s.d.) by the MINOS experiment [32].

The high precision measurements of $\Delta m^2_{\rm A}$, $\tan^2\theta_\odot$ and $\Delta m^2_\odot$ are expected to take place within the next $\sim (6 - 7)$ years. We will assume in what follows that the problem of measuring or tightly constraining $\sin^2\theta$ will also be resolved within the indicated period. Under these conditions, the largest uncertainty in the comparison of the theoretically predicted value of $|<m>|$ with that determined in the $(\beta\beta)_{0\nu}$-decay experiments would be associated with the corresponding $(\beta\beta)_{0\nu}$-decay nuclear matrix elements. We will also assume in what follows that by the time one or more $(\beta\beta)_{0\nu}$-decay experiments of the next generation will be operative (2009 – 2010) at least the physical range of variation of the values of the relevant $(\beta\beta)_{0\nu}$-decay nuclear matrix elements will be unambiguously determined.

Following [25,64], we will parametrize the uncertainty in $|<m>|$ due to the imprecise knowledge of the relevant nuclear matrix elements — we will use the term "theoretical uncertainty" for the latter — through a parameter ζ, $\zeta \geq 1$, defined as:

$$|<m>| = \zeta\Big((|<m>|_{\rm exp})_{\rm MIN} \pm \Delta\Big)\,, \qquad (27)$$

where $(|<m>|_{\rm exp})_{\rm MIN}$ is the value of $|<m>|$ obtained from the measured $(\beta\beta)_{0\nu}$-decay half life-time of a given nucleus using *the largest nuclear matrix element* and Δ is the experimental error. An experiment measuring a $(\beta\beta)_{0\nu}$-decay half-life time will thus determine a range of $|<m>|$ corresponding to

$$(|<m>|_{\rm exp})_{\rm MIN} - \Delta \leq |<m>| \leq \zeta\Big((|<m>|_{\rm exp})_{\rm MIN} + \Delta\Big)\,. \qquad (28)$$

[j] For further details concerning the calculation of the uncertainty in $|<m>|$ in this case see [25,64].

The currently estimated range of ζ^2 for experimentally interesting nuclei varies from 3.5 for ^{48}Ca to 38.7 for ^{130}Te, see, e.g., Table 2 in [47] and [66]. For ^{76}Ge and ^{82}Se it is [47] ~ 10.

In order to be possible to distinguish between the NH and IH spectra, between the NH and QD spectra, and between IH and QD spectra, the following inequalities must hold, respectively:

$$\zeta \, |{<}m{>}|^{\rm NH}_{\rm max} < |{<}m{>}|^{\rm IH}_{\rm min} \,, \tag{29}$$

$$\zeta \, |{<}m{>}|^{\rm NH}_{\rm max} < |{<}m{>}|^{\rm QD}_{\rm min} \,, \tag{30}$$

$$\zeta \, |{<}m{>}|^{\rm IH}_{\rm max} < |{<}m{>}|^{\rm QD}_{\rm min} \,, \ \zeta \geq 1 \,. \tag{31}$$

These conditions imply, as it is not difficult to demonstrate [64], upper limits on $\tan^2\theta_\odot$ which are functions of the neutrino oscillation parameters and of ζ.

In Fig. 3 (taken from [64]) the upper bounds on $\tan^2\theta_\odot$, for which one can distinguish the NH spectrum from the IH spectrum and from that of the QD type, are shown as a function of $\Delta m^2_\odot$ for $\Delta m^2_{\rm A} = 3\times 10^{-3}$ eV^2, $\sin^2\theta = 0.05$ and 0.0 and different values of ζ. For the NH vs IH spectrum results for $\sin^2\theta = 0.01$ are also shown. In the case of the QD spectrum values of $m_0 = 0.2;\ 1.0$ eV are used.

As Fig. 3 demonstrates, the dependence of the maximal value of $\tan^2\theta_\odot$ of interest on m_0 and $\sin^2\theta$ in the NH versus QD case is rather weak. This is not so in what concerns the dependence on $\sin^2\theta$ in the NH versus IH case: the maximal value of $\tan^2\theta_\odot$ under discussion can increase noticeably (e.g., by a factor of $\sim(1.2-1.5)$) when $\sin^2\theta$ decreases from 0.05 to 0. As it follows from Fig. 3, it would be possible to distinguish between the NH and QD spectra for the values of $\tan^2\theta_\odot$ favored by the data for values of $\zeta \cong 3$, or even somewhat bigger than 3. In contrast, the possibility to distinguish between the NH and IH spectra for $\zeta \cong 3$ depends critically on the value of $\sin^2\theta$: as Fig. 3 indicates, this would be possible for the current best fit value of $\tan^2\theta_\odot$ and, e.g., $\Delta m^2_\odot = (5.0-15)\times 10^{-5}$ eV^2, provided $\sin^2\theta \lesssim 0.01$.

In Fig. 4 (taken from [64]) we show the maximal value of $\tan^2\theta_\odot$ permitting to distinguish between the IH and QD spectra as a function of $\Delta m^2_{\rm A}$, for $\sin^2\theta =$ 0.05 and 0.0, $\Delta m^2_\odot = 7.0\times 10^{-5}$ eV^2, $m_0 = 0.2;\ 0.5;\ 1.0$ eV, and $\zeta = 1.0;\ 1.5;\ 2.0;\ 3.0$. The upper bound on $\tan^2\theta_\odot$ of interest depends strongly on the value of m_0. It decreases with the increasing of $\Delta m^2_{\rm A}$. As it follows from Fig. 4, for the values of $\Delta m^2_{\rm A}$ favored by the data and for $\zeta \gtrsim 2$, distinguishing between the IH and QD spectra in the case of $m_0 \cong 0.20$ eV requires too small, from the point of view of the existing data, values of $\tan^2\theta_\odot$. For $m_0 \gtrsim 0.40$ eV, the values of $\tan^2\theta_\odot$ of interest fall in the ranges favored by the solar neutrino and KamLAND data even for $\zeta = 3$.

These quantitative analyses show that if $|{<}m{>}|$ is found to be non-zero in the future $(\beta\beta)_{0\nu}$-decay experiments, it would be easier, in general, to distinguish between the spectrum with NH and those with IH or of QD type using the data on $|{<}m{>}| \neq 0$,

than to distinguish between the IH and the QD spectra. Discriminating between the latter would be less demanding if m_0 is sufficiently large.

5. Constraining the Majorana CP-Violation Phases

The problem of detection of CP-violation in the lepton sector is one of the most formidable and challenging problems in the studies of neutrino mixing. As was noticed in [24], the measurement of $|<\!m\!>|$ alone could exclude the possibility of the two Majorana CP-violation phases α_{21} and α_{31}, present in the PMNS matrix being equal to zero. However, such a measurement cannot rule out without additional input the case of the two phases taking different *CP-conserving* values. The additional input needed for establishing CP-violation could be, e.g., the measurement of neutrino mass $m_{\bar{\nu}_e}$ in ^{3}H β-decay experiment KATRIN [41], or the cosmological determination of the sum of the three neutrino masses [44], $\Sigma = m_1 + m_2 + m_3$, or a derivation of a sufficiently stringent upper limit on Σ. At present no viable alternative to the measurement of $|<\!m\!>|$ for getting information on the Majorana CP-violating phases α_{21} and α_{31} exists, or can be foreseen to exist in the next ~ 8 years.

The possibility to get information on the CP-violation due to the Majorana phases α_{21} and α_{31} by measuring $|<\!m\!>|$ was studied by a large number of authors [20,21,22,23,26], and more recently, e.g., in [27,25]. The authors of [27] took into account in their analysis, in particular, the effect of the uncertainty in the knowledge of the nuclear matrix elements on the measured value of $|<\!m\!>|$. After making a certain number of assumptions about the experimental and theoretical developments in the field of interest that may occur by the year 2020 [k], they claim to have shown "once and for all that it is impossible to detect CP-violation from $(\beta\beta)_{0\nu}$-decay in the foreseeable future." A different approach to the problem was used in [25], where an attempt was made to determine the conditions under which CP-violation might be detected from a measurement of $|<\!m\!>|$ and $m_{\bar{\nu}_e}$ or Σ, or of $|<\!m\!>|$ and a sufficiently stringent upper limit Σ. We will summarize the results obtained in the latter study.

The analysis in [25] is based on prospective input data on $|<\!m\!>|$, $m_{\bar{\nu}_e}$, Σ, $\tan^2\theta_\odot$, etc. The effect of the nuclear matrix element uncertainty was included in the analysis. For example, in the case of the inverted hierarchical spectrum ($m_1 \ll m_2 \simeq m_3$, $m_1 < 0.02$ eV), a "just-CP-violating" region [21] — a value of $|<\!m\!>|$ in this region would signal unambiguously CP-violation in the lepton sector due to Majorana CP-violating phases, would be present if

$$(|<\!m\!>|_{\rm exp})_{\rm MAX} < \sqrt{(\Delta m^2_{\rm atm})_{\rm MIN}} \tag{32}$$

$$(|<\!m\!>|_{\rm exp})_{\rm MIN} > \sqrt{(\Delta m^2_{\rm atm})_{\rm MAX}}\,(\cos 2\theta_\odot)_{\rm MAX}, \tag{33}$$

where $(|<\!m\!>|_{\rm exp})_{\rm MAX(MIN)}$ is the largest (smallest) experimentally allowed value of

[k] It is supposed in [27], in particular, that $|<\!m\!>|$ will be measured with a 25% (1 s.d.) error and that the uncertainty in the $(\beta\beta)_{0\nu}$-decay nuclear matrix elements will be reduced to a factor of 2.

$|<m>|$, taking into account both the experimental error on the measured $(\beta\beta)_{0\nu}$-decay half life-time and the uncertainty due to the evaluation of the nuclear matrix elements. Condition (33) depends crucially on the value of $(\cos 2\theta_\odot)_{\rm MAX}$ and it is less stringent for smaller values of $(\cos 2\theta_\odot)_{\rm MAX}$ [24].

Using the parametrization given in eq. (27), the necessary condition permitting to establish, in principle, that the CP-symmetry is violated due to the Majorana CP-violating phases reads:

$$1 \leq \zeta < \frac{\sqrt{(\Delta m^2_{\rm atm})_{\rm MIN}}}{\sqrt{(\Delta m^2_{\rm atm})_{\rm MAX}}\,(\cos 2\theta_\odot)_{\rm MAX} + 2\Delta}\,. \tag{34}$$

Obviously, the smaller $(\cos 2\theta_\odot)_{\rm MAX}$ and Δ the larger the "theoretical uncertainty" which might allow one to make conclusions concerning the CP-violation of interest.

A similar analysis was performed in the case of QD neutrinos mass spectrum. The results can be summarized as follows. The possibility of establishing that the Majorana phases α_{21} and α_{31} have CP-nonconserving values requires quite accurate measurements of $|<m>|$ and, say, of $m_{\bar{\nu}_e}$ or Σ, and holds only for a limited range of values of the relevant parameters. More specifically, proving that CP-violation associated with Majorana neutrinos takes place requires, in particular, a relative experimental error on the measured value of $|<m>|$ not bigger than (15 – 20)%, a "theoretical uncertainty" in the value of $|<m>|$ due to an imprecise knowledge of the corresponding nuclear matrix elements smaller than a factor of 2, a value of $\tan^2\theta_\odot \gtrsim 0.55$, and values of the relevant Majorana CP-violating phases (α_{21}, α_{32}) typically within the ranges of $\sim (\pi/2 - 3\pi/4)$ and $\sim (5\pi/4 - 3\pi/2)$.

6. Conclusions

Future $(\beta\beta)_{0\nu}-$decay experiments have a remarkable physics potential. They can establish the Majorana nature of the neutrinos with definite mass ν_j. If the latter are Majorana particles, the $(\beta\beta)_{0\nu}-$decay experiments can determine the type of the neutrino mass spectrum and can provide unique information on the absolute scale of neutrino masses. They can also provide unique information on the Majorana CP-violation phases present in the PMNS neutrino mixing matrix. The knowledge of the values of the relevant $(\beta\beta)_{0\nu}-$decay nuclear matrix elements with a sufficiently small uncertainty is crucial for obtaining quantitative information on the neutrino mass and mixing parameters from a measurement of $(\beta\beta)_{0\nu}-$decay half life-time.

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Reference	$(\tan^2\theta_\odot)_{\rm BF}$	$(\Delta m^2_\odot)_{\rm BF}$	$\lvert<m>\rvert^{\rm NH}_{\rm max}$	$\lvert<m>\rvert^{\rm IH}_{\rm min}$	$\lvert<m>\rvert^{\rm QD}_{\rm min}$
[9]	0.46	7.3	5.9 (3.9)	18.4	59.9
[10]	0.42	7.2	5.7 (3.7)	20.3	67.2
[11]	0.43	7.0	5.7 (3.7)	19.8	65.3

Table 1: The best-fit values of $\tan^2\theta_\odot$ and $\Delta m^2_\odot$ (in units of 10^{-5} eV^2) in the LMA solution region, as reported by different authors. Given are also the calculated maximal values of $\lvert<m>\rvert$ (in units of 10^{-3} eV) for the NH spectrum and the minimal values of $\lvert<m>\rvert$ (in units of 10^{-3} eV) for the IH and QD spectra. The results for $\lvert<m>\rvert$ in the cases of NH and IH spectra are obtained for $m_1 = 10^{-3}$ eV and the best-fit value of $\Delta m^2_{\rm A}$, $\Delta m^2_{\rm A} = 2.7\times10^{-3}$ eV ^{2} [12], while those for the QD spectrum are derived for $m_0 = 0.2$ eV. In all cases $\sin^2\theta = 0.05$ has been used. For $\lvert<m>\rvert^{\rm NH}_{\rm max}$ we included in brackets also the values for $\sin^2\theta = 0.01$. The chosen value of $\Delta m^2_{\rm A}$ corresponds to $\lvert<m>\rvert^{\rm IH}_{\rm max} = 52.0\times10^{-3}$ eV. (From [64].)

Reference	$\tan^2\theta_\odot$	$\Delta m^2_\odot$	$\lvert<m>\rvert^{\rm NH}_{\rm max}$	$\lvert<m>\rvert^{\rm IH}_{\rm min}$	$\lvert<m>\rvert^{\rm QD}_{\rm min}$
[9]	0.32 − 0.72	5.6 − 17	8.6 (6.6)	7.6	20.6
[10]	0.31 − 0.56	6.0 − 8.7	6.6 (4.5)	13.0	43.2
[11]	0.31 − 0.66	5.9 − 8.9	7.0 (4.9)	9.5	28.6

Table 2: The ranges of allowed values of $\tan^2\theta_\odot$ and $\Delta m^2_\odot$ (in units of 10^{-5} eV^2) in the LMA solution region, obtained at 90% C.L. by different authors. Given are also the corresponding maximal values of $\lvert<m>\rvert$ (in units of 10^{-3} eV) for the NH spectrum, and the minimal values of $\lvert<m>\rvert$ (in units of 10^{-3} eV) for the IH and QD spectra. The results for the NH and IH spectra are obtained for $m_1 = 10^{-3}$ eV, while those for the QD spectrum correspond to $m_0 = 0.2$ eV. $\Delta m^2_{\rm A}$ was assumed to lie in the interval [12] $(2.3 - 3.1)\times10^{-3}$ eV ^{2}. This implies $\lvert<m>\rvert^{\rm IH}_{\rm max} = 55.7\times10^{-3}$ eV. As in Table 1, in all cases $\sin^2\theta = 0.05$ has been used. For $\lvert<m>\rvert^{\rm NH}_{\rm max}$ we included in brackets also the values for $\sin^2\theta = 0.01$. (From [64].)

Reference	$\lvert<m>\rvert^{\rm NH}_{\rm max}\,(s^2 = 0.05)$	$\lvert<m>\rvert^{\rm NH}_{\rm max}\,(s^2 = 0.01)$	$\lvert<m>\rvert^{\rm IH}_{\rm min}$	$\lvert<m>\rvert^{\rm QD}_{\rm min}$
[9]	6.1 (6.7)	4.1 (4.4)	16.5 (12.9)	55.9 (48.2)
[10]	6.0 (6.5)	3.9 (4.2)	18.3 (14.6)	63.3 (55.9)
[11]	6.0 (6.5)	3.9 (4.2)	17.9 (14.1)	61.4 (54.0)

Table 3: The values of $\lvert<m>\rvert^{\rm NH}_{\rm max}$, $\lvert<m>\rvert^{\rm IH}_{\rm min}$ and $\lvert<m>\rvert^{\rm QD}_{\rm min}$ (in units of 10^{-3} eV), calculated using the best-fit values of solar and atmospheric neutrino oscillation parameters from Table 1 and including 1 s.d. (3 s.d) uncertainties of 5 % (15%) on $\tan^2\theta_\odot$ and $\Delta m^2_\odot$, and of 10 % (30%) on $\Delta m^2_{\rm A}$. In this case one has: $\lvert<m>\rvert^{\rm IH}_{\rm max} = 54.5\ (59.2)\times10^{-3}$ eV. (From [64].)

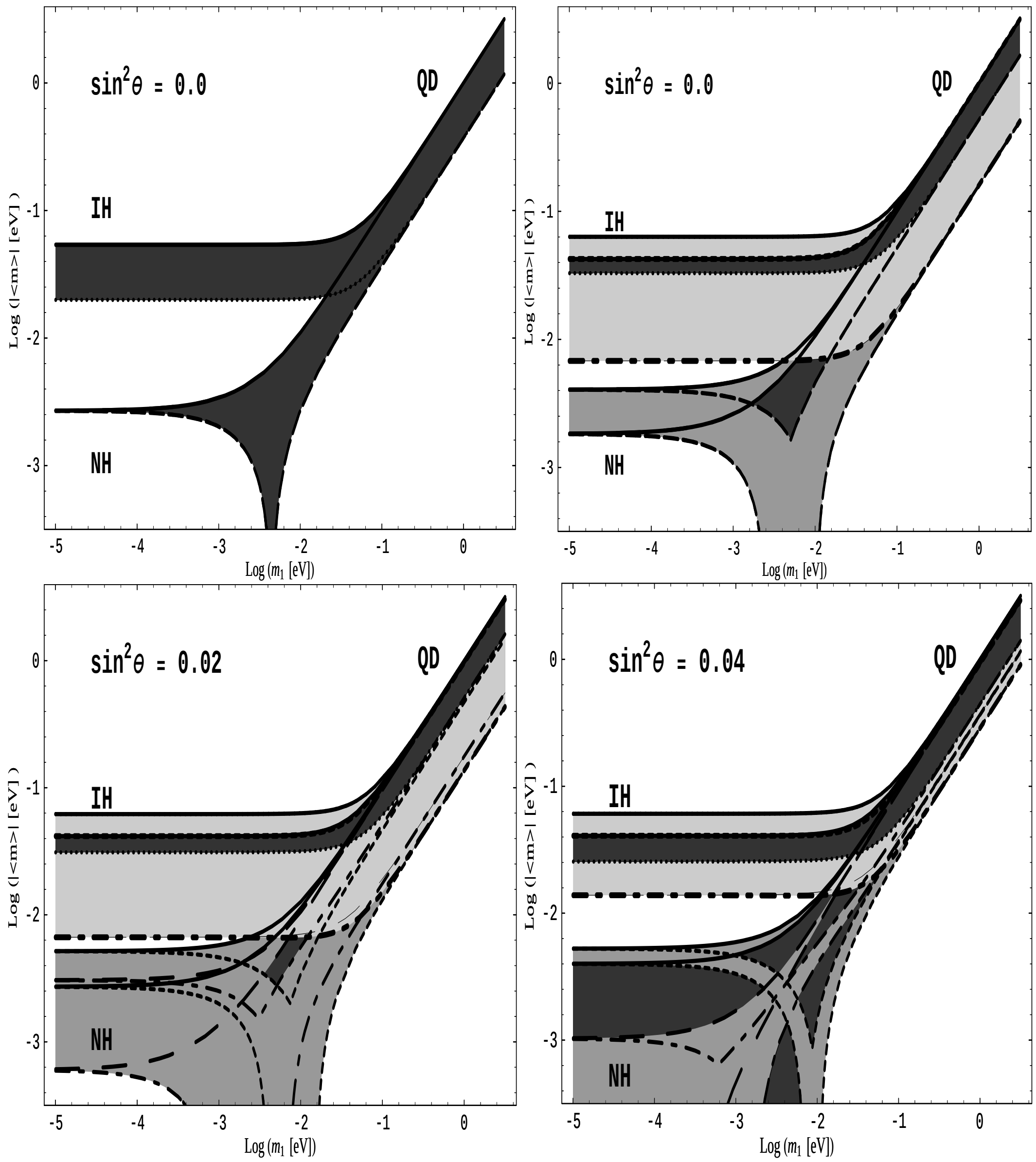

Figure 1: The dependence of $|<m>|$ on m_1 for the solution LMA-I, $\Delta m^2_\odot = \Delta m^2_{21}$ and $\Delta m^2_\odot = \Delta m^2_{32}$, and for the best fit (upper left panel), and the 90% C.L. allowed (upper right and lower panels), values of the neutrino oscillation parameters found in refs. [9,12]. The values of $\sin^2\theta$ used are 0.0 (upper panels), 0.02 (lower left panel) and 0.04 (lower right panel). In the case of CP-conservation, $|<m>|$ takes values: i) for the upper left panel and $\Delta m^2_\odot = \Delta m^2_{21}$ ($\Delta m^2_\odot = \Delta m^2_{32}$) on *a)* the lower (upper) solid line if $\eta_{21(32)} = 1$ and $\eta_{31(21)} = \pm 1$, *b)* the long-dashed (dotted) line if $\eta_{21(32)} = -1$ and $\eta_{31(21)} = \pm 1$; ii) for the upper right panel and $\Delta m^2_\odot = \Delta m^2_{21}$ ($\Delta m^2_\odot = \Delta m^2_{32}$) - in the medium grey (light grey) regions *a)* between the two lower solid lines (the upper solid line and the short-dashed line) if $\eta_{21(32)} = 1$ and $\eta_{31(21)} = \pm 1$, *b)* between the two long-dashed lines (the dotted and the dash-dotted lines) if $\eta_{21(32)} = -1$ and $\eta_{31(21)} = \pm 1$; for the two lower panels and $\Delta m^2_\odot = \Delta m^2_{21}$ - in the medium grey regions *a)* between the two lower solid lines if $\eta_{21} = \eta_{31} = 1$, *b)* between the long-dashed lines if $\eta_{21} = -\eta_{31} = 1$, *c)* between the two lower dash-dotted lines if $\eta_{21} = -\eta_{31} = -1$, *d)* between the two lower short-dashed lines if $\eta_{21} = \eta_{31} = -1$; and iii) for the two lower panels and $\Delta m^2_\odot = \Delta m^2_{32}$ - in the light grey regions delimited *a)* by the upper solid and the upper short-dashed lines if $\eta_{32} = \pm\eta_{21} = 1$, *b)* by the dotted and the upper dash-dotted lines if $\eta_{32} = \pm\eta_{21} = -1$. Values of $|<m>|$ in the dark grey regions signal CP-violation.

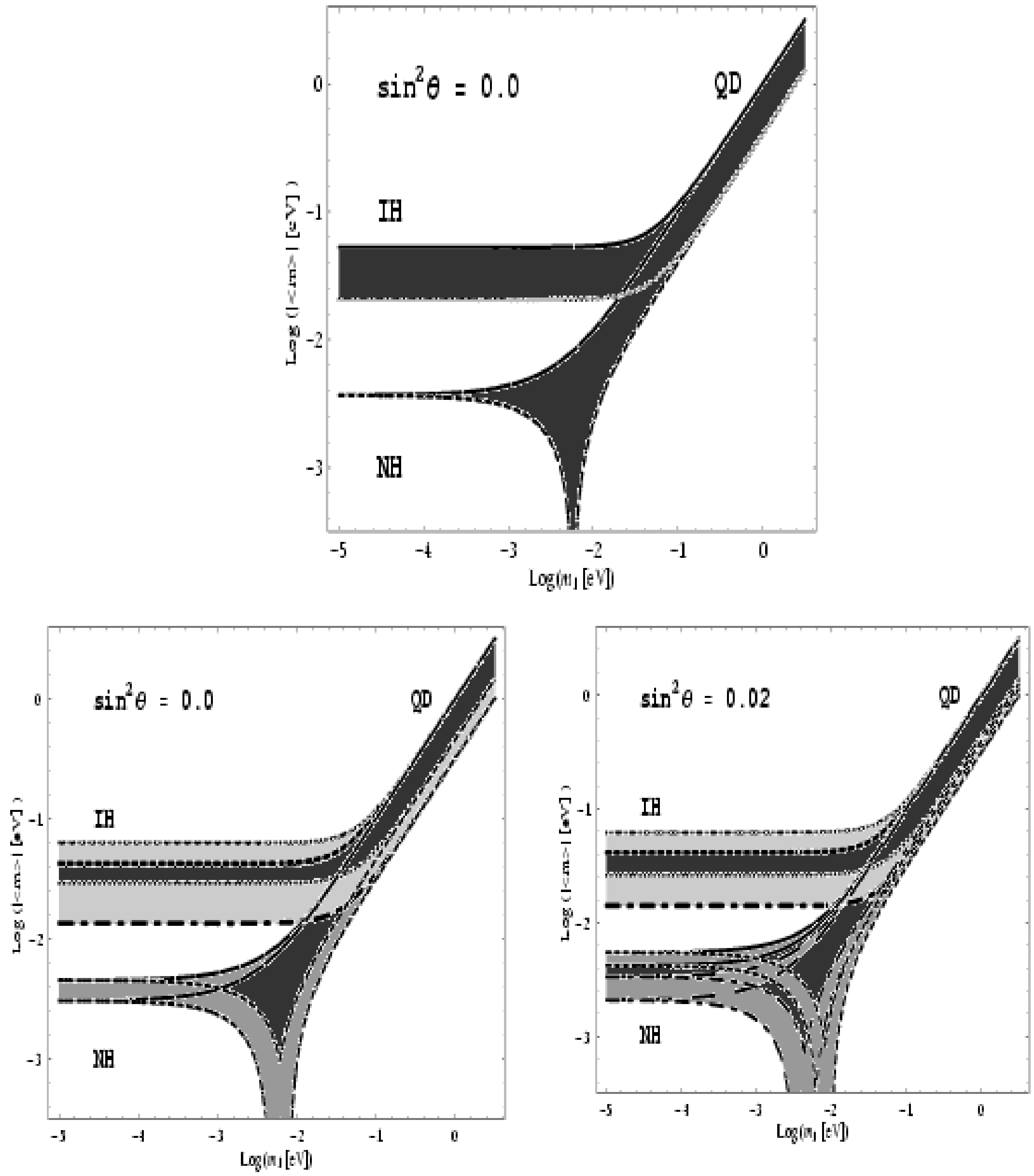

Figure 2: The dependence of $|<m>|$ on m_1 in the case of the solution LMA-II, for $\Delta m^2_\odot = \Delta m^2_{21}$ and $\Delta m^2_\odot = \Delta m^2_{32}$, and for the best fit values (upper panel) and the 90% C.L. allowed values (lower panels) of the neutrino oscillation parameters found in refs. [9,12]. The value of $\sin^2\theta$ used are 0.0 (upper and lower left panels) and 0.02 (lower right panel). In the case of CP-conservation, the allowed values of $|<m>|$ are constrained to lie on the same lines and regions as in Fig. 1: for the upper (lower left) panel see the description of the upper left (upper right) panel in Fig. 1, and for the lower right panel refer to the explanations for the two lower panels in Fig. 1. Values of $|<m>|$ in the dark grey regions signal CP-violation.

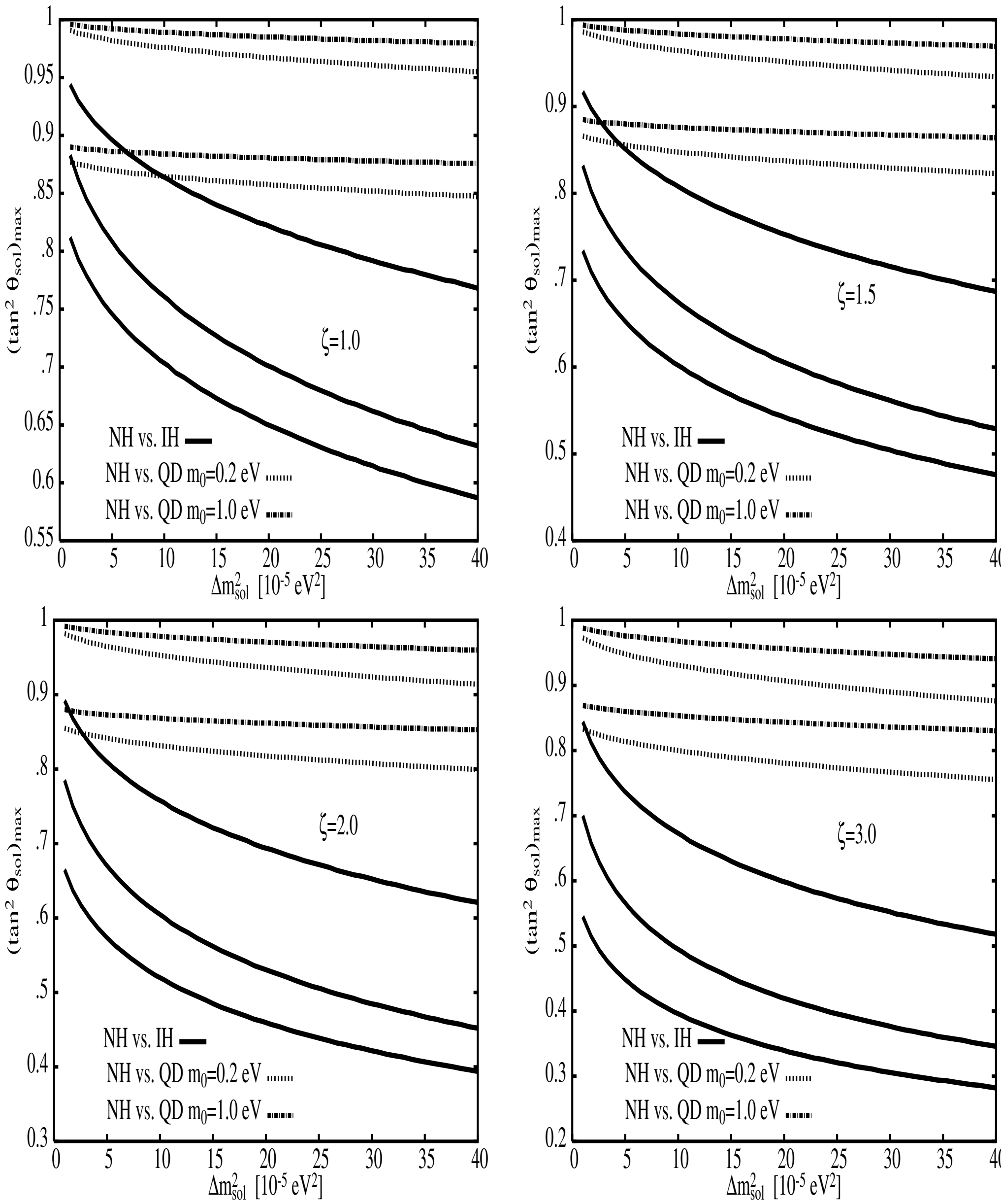

Figure 3: The upper bound on $\tan^2\theta_\odot$, for which one can distinguish the NH spectrum from the IH spectrum and from that of QD type, as a function of $\Delta m^2_\odot$ for $\Delta m^2_{\rm A} = 3\times10^{-3}$ eV ^{2} and different values of ζ. The lower (upper) line corresponds to $\sin^2\theta = 0.05$ (0). For NH vs. IH there is a third (middle) line corresponding to $\sin^2\theta = 0.01$. (From [64].)

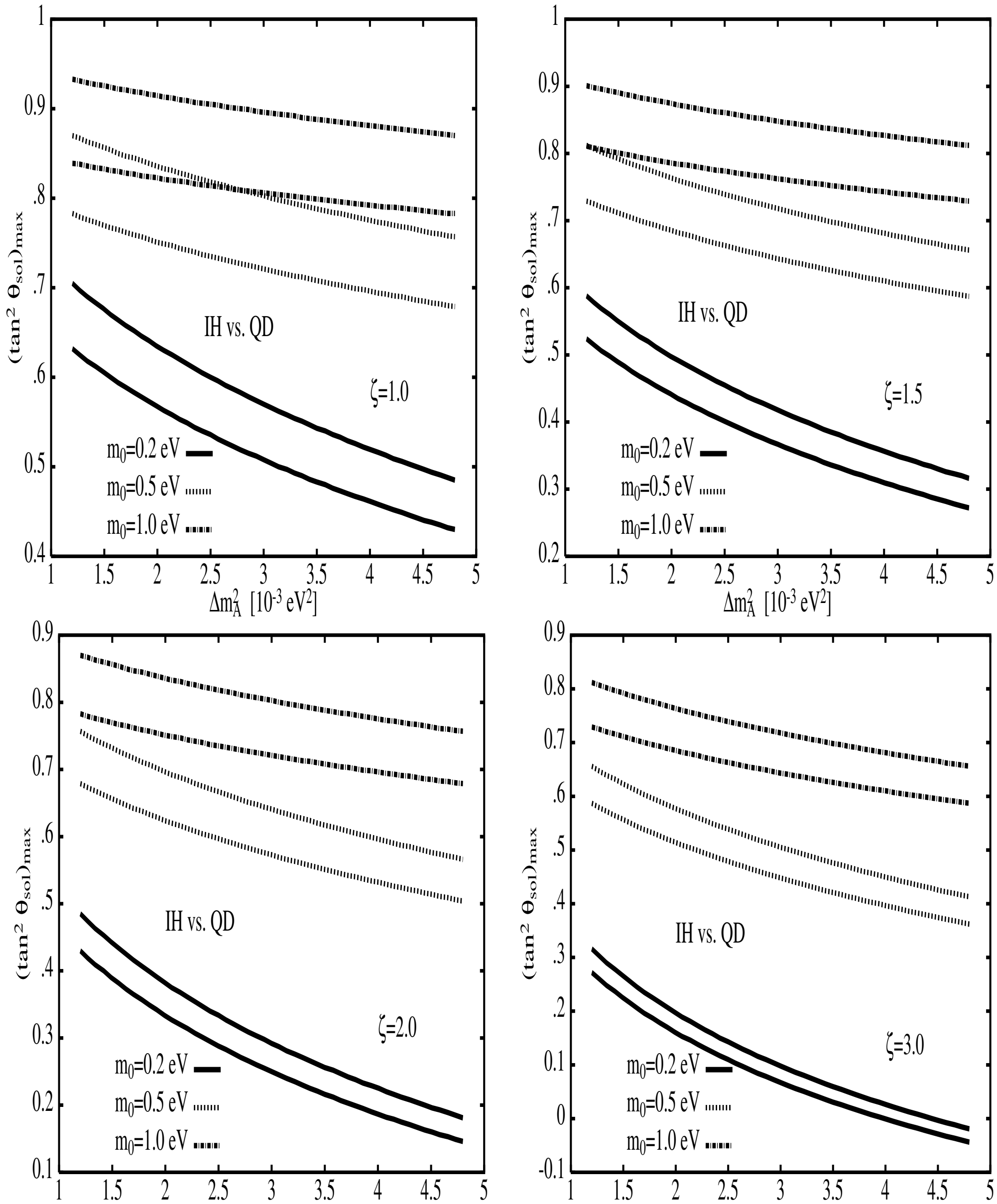

Figure 4: The upper bound on $\tan^2\theta_\odot$ allowing one to discriminate between the IH and the QD neutrino mass spectra, as a function of $\Delta m^2_{\rm A}$ for different values of ζ. The lower (upper) line corresponds to $\sin^2\theta = 0.05$ (0). (From [64].)